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Mapping of Stochastic Dynamics onto Associated Quantum Models and ($d+1$)-Dimensional Classical and Static Systems

Toni Schneider[1]

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Much effort has been devoted to the subjects of dynamic,[1] quantum,[2] and classical critical phenomena,[3] although these research fields developed rather independently. Recently, however, remarkable connections have been established,[4–9] achieved by using the connection between the time-dependent Ginzburg–Landau equation (TDGL) in d dimensions, describing critical dynamics, and the Fokker–Planck equation. The latter is then reduced to an imaginary-time Schrödinger equation, defining the Hamiltonian of the quantum system, which in turn can be mapped onto a static and classical ($d+1$)-dimensional counterpart.[4,5] Another interesting aspect of these mappings is the simulation of quantum systems in terms of Langevin equations[4,5,10] and the construction of quantum systems with soluble ground-state expectative values.[9] Up to now, and as far as critical phenomena are concerned, these general relationships have been used to derive the following results: (i) dynamic scaling was traced back to anisotropic scaling in an associated ($d+1$)-dimensional classical and static model[5,6]; (ii) dynamic critical exponents were calculated with conventional renormalization-group techniques from the ($d+1$)-dimensional classical and static counterpart[6]; (iii) the equivalence of the real-space renormalization group of critical dynamics and of the real-space renormalization group for quantum systems was established; and (iv) the ($d+1$)-dimensional static and classical model resulting from the TDGL system was shown to exhibit a tricritical Lifshitz point (TLP) belonging to

[1] IBM Zurich Research Laboratory, Säumerstr. 4, 8803 Rüschlikon, Switzerland.

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a novel class of TLPs, which result from a relevant, nonlocal quartic field interaction previously ignored.[8] We also note that the stochastic dynamics of a d-dimensional model can be mapped directly onto the statics of a classical $(d+1)$-one.[11,12]

REFERENCES

1. P. C. Hohenberg and B. I. Halperin, *Rev. Mod. Phys.* **49**:435 (1977).
2. R. Morf, T. Schneider, and E. Stoll, *Phys. Rev. B* **16**:462 (1977).
3. S. K. Ma, *Modern Theory of Critical Phenomena* (Benjamin, Reading, Massachusetts, 1976).
4. T. Schneider, M. Zannetti, R. Badii, and H. R. Jauslin, *Phys . Rev. Lett.* **53**:2191 (1984).
5. T. Schneider, M. Zannetti, and R. Badii, *Phys. Rev. B* **31**:2941 (1985).
6. T. Schneider and M. Schwartz, *Phys. Rev. B* **31**:7484 (1985).
7. T. Schneider and M. Zannetti, *Phys. Rev. B* **32**:6108 (1985).
8. A. Aharony, E. Domany, R. M. Hornreich, T. Schneider, and M. Zannetti, *Phys. Rev. B* **32**:3358 (1985).
9. T. Schneider and R. Badii, *J. Phys. A* **18**:L187 (1985).
10. T. Schneider, R. Badii, and P. Sörensen, to be published.
11. C. M. Bender, F. Cooper, and B. Freedman, *Nucl. Phys . B* **219**:61 (1983).
12. E. Domany and J. E. Grubernahs, *Phys. Rev. B* **34**:3354 (1985).